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Thermal Effects on Saxion in Supersymmetric Model with Peccei-Quinn Symmetry

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Abstract

We consider supersymmetric model with Peccei-Quinn symmetry and study effects of saxion on the evolution of the universe, paying particular attention to the effects of thermal bath. The axion multiplet inevitably couples to colored particles, which induces various thermal effects. In particular, (i) saxion potential is deformed by thermal effects, and (ii) coherent oscillation of the saxion dissipates via the interaction with hot plasma. These may significantly affect the evolution of the saxion in the early universe.

1 Introduction

Peccei-Quinn (PQ) mechanism [1, 2] provides an elegant solution to the strong CP problem. Promoting the θ -parameter to a dynamical variable, $\theta = 0$ is realized after the spontaneous breaking of the PQ symmetry. In addition, in such a framework, a very light scalar field, called axion, shows up [3, 4]. The coherent oscillation of the axion field is a viable candidate of dark matter of our universe if the PQ symmetry breaking scale is around 10^{12} GeV [5]. Thus, the PQ mechanism has been attracted many attentions not only from particle-physics point of view but also from cosmology point of view.

If we embed the PQ mechanism into the framework of supersymmetry (SUSY), which is a prominent candidate of the model beyond the standard model, serious cosmological difficulties may show up. In particular, because of the supersymmetry, there should exist superpartners of the axion, i.e., axino and saxion. Thermally produced axino and saxion, as well as coherent oscillation of saxion, may significantly affect the evolution of the universe. In order to construct cosmologically viable supersymmetric PQ model, it is important to understand how the axion multiplet affects the thermal history of the universe.

The purpose of this paper is to study the effect of the saxion on cosmology, paying particular attention to the thermal effects on properties of the saxion. In hadronic axion models [6, 7], axion multiplet couples to extra colored multiplets, which include colored fermions (called PQ fermions), to solve the strong CP problem. Even in DFSZ-type model [8, 9] in which the ordinary Higgses have non-vanishing PQ charges, extra colored multiplets are likely to be introduced to avoid domain-wall problem. The saxion acquires thermal mass of the order of yT (with y being coupling constant) when the cosmic temperature T is higher than the mass of PQ fermions. Even if the temperature is lower so that the densities of the PQ fermions (and their scalar partners) are negligibly small, thermal effect induces logarithmic term in saxion potential [10]. These thermal effects shift the minimum of the potential, which may result in too large amplitude of the coherent oscillation of the saxion. Furthermore, the coherent oscillation of the saxion may dissipate via the interaction with hot QCD plasma. Taking these effects into account, we study cosmological history of a SUSY PQ model.

2 Thermal Effects on Saxion

In supersymmetric PQ model, there exists supermultiplet $\hat{\mathcal{A}}$ (called “axion supermultiplet”) which includes the axion field. Saxion (denoted as σ) is a real scalar field in the axion multiplet:

$$\hat{\mathcal{A}} = \frac{1}{\sqrt{2}}(\sigma + ia) + \sqrt{2}\theta\tilde{a} + (F\text{-term}), \quad (2.1)$$

where a and \tilde{a} are axion and axino, respectively. (Here and hereafter, the “hat” is for superfields.) The saxion potential is lifted only by the supersymmetry breaking effects, and hence is very flat at $T = 0$.

The axion multiplet couples to colored multiplets, which include PQ fermions, to solve the strong CP problem. In this study, we consider the case that the masses of PQ fermions are related to the amplitude of σ . The PQ fermions become massless when σ takes a particular value. By integrating out PQ fermions, the axion multiplet couples to gauge multiplets (in particular, that of $SU(3)_C$) as

$$\mathcal{L}_{\text{int}} = \frac{\alpha_3}{8\pi F_a} \int d^2\theta \hat{\mathcal{A}} \hat{\mathcal{W}}^\alpha \hat{\mathcal{W}}_\alpha + \text{h.c.}, \quad (2.2)$$

where $\hat{\mathcal{W}}$ is the field strength supermultiplet of $SU(3)_C$. (Here and hereafter, $\alpha_i = g_i^2/4\pi$, with g_i being the gauge coupling constant of $U(1)_Y$, $SU(2)_L$, $SU(3)_C$ for $i = 1, 2$, and 3 , respectively)

The potential of the saxion is affected by particles in thermal bath (in particular, by those with $SU(3)_C$ quantum numbers). In addition, scattering processes may also become important. In the following, we discuss possible thermal effects on the saxion.

2.1 Thermal Potential

The first subject is thermal effects on the potential (more precisely, the free energy). The PQ fermions (and their superpartners), which we call Q , affect the potential as

$$V_T(\sigma) = V_0(\sigma) + g_Q^{(\text{B})} V_T^{(\text{B})}(m_Q(\sigma)) + g_Q^{(\text{F})} V_T^{(\text{F})}(m_Q(\sigma)), \quad (2.3)$$

where V_0 is the zero-temperature potential while $V_T^{(\text{B})}$ and $V_T^{(\text{F})}$ are contributions of bosonic and fermionic fields, respectively, and $m_Q(\sigma)$ is the mass of the fields which couple to σ . (Extra contributions exist if particles other than PQ fermions couple to σ .) At the one-loop level, the functions $V_T^{(\text{B/F})}$ are given by [11]

$$V_T^{(\text{B/F})}(M) = \pm \frac{T^4}{\pi^2} \int_0^\infty dz z^2 \ln \left[1 \mp e^{-\sqrt{z^2 + M^2/T^2}} \right], \quad (2.4)$$

where upper and lower signs are for $V_T^{(\text{B})}$ and $V_T^{(\text{F})}$, respectively. In addition, $g_Q^{(\text{B/F})}$ are the number of degrees of freedom. ($g_Q^{(\text{B})} = 1$ for one complex scalar field, and $g_Q^{(\text{F})} = 1$ for one chiral fermion.)

$V_T^{(\text{B})}$ and $V_T^{(\text{F})}$ change their behavior at $M \sim T$. When $M \lesssim T$, $V_T^{(\text{B})}$ and $V_T^{(\text{F})}$ are well approximated as

$$V_T^{(\text{B})}(M) \simeq -\frac{\pi^2}{45} T^4 + \frac{1}{12} T^2 M^2 + O(TM^3), \quad (2.5)$$

$$V_T^{(\text{F})}(M) \simeq -\frac{7\pi^2}{360} T^4 + \frac{1}{24} T^2 M^2 + O(M^4), \quad (2.6)$$

and hence the so-called thermal mass shows up. When σ satisfies the condition $m_Q(\sigma) \lesssim T$, $V_T(\sigma)$ acquires a new term of $\sim T^2 m_Q^2(\sigma)$. For $M \gtrsim T$, on the contrary, $V_T^{(\text{B})}$ and $V_T^{(\text{F})}$ rapidly go to zero.

If the temperature is high enough so that $T \gg m_Q(\sigma_0)$, where σ_0 is the zero-temperature vacuum expectation value (VEV) of σ , the PQ fermions are in thermal bath as relativistic particles even when $\sigma \sim \sigma_0$. In such a case, the expectation value of σ is determined by the thermal mass term. Because $V_T^{(B)}(m_Q(\sigma))$ and $V_T^{(F)}(m_Q(\sigma))$ are minimized at $\sigma = \sigma_Q$ (where σ_Q satisfies $m_Q(\sigma_Q) = 0$), we expect only one minimum of the potential at $\sigma \sim \sigma_Q$ at high enough temperature. The situation changes when the temperature is lower than $m_Q(\sigma_0)$. In such a case, the number densities of Q are Boltzmann suppressed at $\sigma \sim \sigma_0$. Thermal effects are negligible around the zero-temperature minimum and there exist a minimum at $\sigma \sim \sigma_0$. (Effect of the logarithmic thermal correction to the potential [10], which affects the position of the minimum, may be sizable. See the discussion below.) On the other hand, if $V_0(\sigma)$ is flat enough, the minimum at $\sigma \sim \sigma_Q$ also remains. In such a case, there are two distinctive minima of the potential; even though there exists a minimum at $\sigma \sim \sigma_0$, the scalar field σ can be thermally trapped in the different minimum close to the symmetry enhanced point. Then, the minimum at $\sigma \sim \sigma_Q$ disappears when the temperature is so low that the shape of the potential around $\sigma \sim \sigma_Q$ is determined by the zero-temperature potential.

The saxion potential has another important term which is not taken into account in Eq. (2.3). Because the free energy of hot QCD plasma has a contribution proportional to $g_3^2(T)T^4$ [12], and also because $g_3^2(T)$ depends on $m_Q(\sigma)$ (i.e., the mass of colored particles) if $m_Q(\sigma) > T$, the free energy has the following term [10]^{#1}

$$V_L(\sigma) \equiv a_L \alpha_3^2(T) T^4 \ln |m_Q(\sigma)|^2, \quad (2.7)$$

where a_L is a constant a bit larger than 1.

2.2 Scattering Processes and Dissipation

Next, we consider the scattering processes and dissipation.

It has been known that scatterings of thermal particles contribute to the production processes of axion, axino and saxion. Define the yield variable as

$$Y_X \equiv \frac{n_X}{s}, \quad (2.8)$$

where n_X is the number density of particle X while $s = \frac{45}{2\pi^2} g_*(T) T^3$ is the entropy density (with g_* being the effective number of massless degrees of freedom). Then, if the interaction of the axion multiplet with the minimal supersymmetric standard model (MSSM) particles is dominated by the operator given in Eq. (2.2), thermally produced axino abundance in radiation-dominated era is given by [13]

$$\left[Y_{\tilde{a}}^{(\text{th})} \right]_{\text{RD}} \simeq \min \left[Y_{\tilde{a}}^{(\text{eq})}, 0.20 \times \alpha_3^3 \ln \left(\frac{0.0977}{\alpha_3} \right) \left(\frac{T_{\text{R}}}{10^7 \text{ GeV}} \right) \left(\frac{F_a}{10^{11} \text{ GeV}} \right)^{-2} \right], \quad (2.9)$$

^{#1}The logarithmic term is expected to disappear in the region where $m_Q(\sigma)$ is smaller than $\sim T$. In such a region, however, $V_T^{(B)}$ and $V_T^{(F)}$ dominate, so the effect of the logarithmic term is not important.

where T_R is the reheating temperature, and $Y_a^{(\text{eq})} \simeq 1.8 \times 10^{-3}$ is the thermal abundance of axino. A precise calculation of the abundance of thermally produced saxion is not available yet. Here, using the fact that the production processes of saxion and axino are governed by the same supersymmetric interaction, we approximate the thermally produced saxion abundance as $[Y_\sigma^{(\text{th})}]_{\text{RD}} \simeq \frac{2}{3}[Y_{\tilde{a}}^{(\text{th})}]_{\text{RD}}$.

The thermally produced axino may be cosmologically harmful [14]. If it is the lightest superparticle (LSP) and is stable, it contributes to the present dark matter density. In such a case, a very stringent upper bound on the reheating temperature is obtained in order not to overproduce axino. (For recent discussion, see, for example, [15, 16] and references therein.) Even if the saxion is unstable, the LSP produced by the decay of axino contributes to the present mass density of the universe. If all the LSPs produced by the axino decay survive until today, low reheating temperature is needed to avoid the overproduction of the LSP. In the case of unstable axino, these cosmological difficulties can be avoided if the LSP has large enough pair annihilation cross section [17]. This may be the case if, for example, the LSP is neutral Wino [18, 19, 20]. In addition, R -parity violation may be another possibility to avoid the cosmological difficulty.

Existence of hot plasma not only increases but also reduces the number density of particles in the PQ sector. Such an effect is particularly important in studying the evolution of saxion oscillation. The dissipation rate of the saxion is related to the bulk viscosity of hot plasma [21, 22], and is estimated as^{#2}

$$\Gamma_{\text{diss}} \sim \frac{9\alpha_3^2}{128\pi^2 \ln \alpha_3^{-1}} \frac{T^3}{F_a^2}, \quad (2.10)$$

where, if the saxion is far away from the minimum, F_a should be replaced by the effective axion decay constant $F_a^{(\text{eff})}$ (which is $\sim \sigma$). Notice that, if σ interacts with particles other than MSSM gauge multiplets, Γ_{diss} receives contributions from those extra particles. We will see that this happens in some case.

The dissipation rate Γ_{diss} is an important quantity in studying the saxion oscillation in the early universe because Γ_{diss} may become larger than the expansion rate of the universe H . For the case that $\Gamma_{\text{diss}} \ll H$, the saxion oscillates if the initial amplitude is non-vanishing. Then, with parabolic potential, for example, the energy density of the saxion decreases as a^{-3} with the cosmic expansion (where a is the scale factor). On the contrary, if Γ_{diss} is sizable, we cannot neglect the effect of dissipation. In particular, if $\Gamma_{\text{diss}} \gtrsim H$, coherent oscillation of the saxion dissipates within a time scale shorter than the cosmic time. In radiation dominated universe with $g_*(T_R) = 228.75$, Γ_{diss} given in Eq. (2.10) becomes larger than H when $T \gtrsim 3 \times 10^5$ GeV (5×10^7 GeV, 7×10^9 GeV, 1×10^{12} GeV) for $F_a = 10^9$ GeV (10^{10} GeV, 10^{11} GeV, 10^{12} GeV). One should note that, if $\Gamma_{\text{diss}} \gtrsim H$, $Y_\sigma^{(\text{th})}$ becomes comparable to $Y_\sigma^{(\text{eq})}$. Thus, in such a case, the particles in the axion multiplet are fully thermalized.

Cosmology of models with saxion was considered in the framework in which the saxion field plays the role of the flaton field for thermal inflation [23, 24, 25]. We will consider a

^{#2}The authors thank K. Mukaida for helpful discussion on this issue.

different class of models, taking account of the thermal effects discussed above. We will see that the evolution of the saxion in the early universe can be significantly affected by these effects.

3 Explicit Example

3.1 Model

To see the thermal effects on the evolution of the saxion, we consider the supersymmetric PQ model with the following superpotential:

$$W = W_{\text{MSSM}} + \lambda \hat{S}(\hat{X}\hat{X} - f^2) + y\hat{X}\hat{Q}_0\hat{Q}_+, \quad (3.1)$$

where W_{MSSM} is the superpotential of the MSSM, and \hat{X} (+1), \hat{X} (-1), \hat{Q}_0 (0), \hat{Q}_+ (+1), and \hat{S} (0) are chiral superfields. (We denote the $U(1)_{\text{PQ}}$ -charge in the parentheses.) \hat{Q}_0 and \hat{Q}_+ are in the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations of $SU(3)_C$; here one pair of PQ fermions are introduced. One combination of \hat{X} and \hat{X} becomes the axion supermultiplet. In addition, y , λ , and f are constants which are chosen to be real and positive.

In the minimum of the supersymmetric scalar potential, we obtain the relation $\bar{X}X = f^2$. Such a constraint eliminates one combination of \hat{X} and \hat{X} from the low-energy spectrum and \hat{X} and \hat{X} are decomposed into the axion multiplet $\hat{\mathcal{A}}$ and heavy multiplet which we call $\hat{\mathcal{X}}$. Without SUSY breaking terms, the relative size of \bar{X} and X is undetermined; such a flat direction corresponds to $\hat{\mathcal{A}}$. The SUSY breaking terms relevant for our study are^{#3}

$$V_{\text{soft}} = m_1^2|X|^2 + m_2^2|\bar{X}|^2 + m_S^2|S|^2, \quad (3.2)$$

where we expect that m_1 , m_2 , and m_S are of the order of the gravitino mass $m_{3/2}$. With the SUSY breaking terms, the VEVs of scalar fields are fixed.

At the minimum of the potential, the axion multiplet $\hat{\mathcal{A}}$ and the heavy multiplet $\hat{\mathcal{X}}$ are embedded into \hat{X} and \hat{X} as

$$\hat{X} = F_a \cos \beta_X - \hat{\mathcal{A}} \cos \beta_X + \hat{\mathcal{X}} \sin \beta_X, \quad (3.3)$$

$$\hat{X} = F_a \sin \beta_X + \hat{\mathcal{A}} \sin \beta_X + \hat{\mathcal{X}} \cos \beta_X, \quad (3.4)$$

where F_a is the axion decay constant which is given by

$$F_a^2 = \langle \bar{X} \rangle^2 + \langle X \rangle^2 = \frac{2f^2}{\sin 2\beta_X}, \quad (3.5)$$

with

$$\cos 2\beta_X \equiv \xi \equiv \frac{m_1^2 - m_2^2}{m_1^2 + m_2^2}. \quad (3.6)$$

^{#3}The tri-linear interaction $S\bar{X}X$ and linear term in S may also exist in V_{soft} , with which the axino mass is generated. In the present model, the axino mass is at most $\frac{1}{\sqrt{2}}m_\sigma$.

By integrating out \hat{Q}_0 and \hat{Q}_+ , we obtain the interaction given in Eq. (2.2). Then, the saxion mass is $m_\sigma = 2m_1m_2/\sqrt{m_1^2 + m_2^2}$.

We note here that, in the present setup, the decay rate of saxion Γ_σ is governed by the ξ parameter defined in Eq. (3.6). In hadronic axion model in which axion multiplet does not couple to the Higgs multiplet, σ dominantly decays into axion pair; decay rate of such a process is given by

$$\Gamma_{\sigma \rightarrow aa} = \frac{\xi^2}{64\pi} \frac{m_\sigma^3}{F_a^2}. \quad (3.7)$$

The saxion also decays into gauge-boson pairs (in particular, the gluon pair) via the interaction given in Eq. (2.2). However, the saxion-gluon-gluon interaction is loop suppressed and hence such a process is expected to be subdominant (as far as $\xi \sim 1$); indeed, the decay rate for the process $\sigma \rightarrow gg$ (with g being the gluon) is given by

$$\Gamma_{\sigma \rightarrow gg} = \frac{8\alpha_3^2}{256\pi^3} \frac{m_\sigma^3}{F_a^2}. \quad (3.8)$$

Numerically, $\Gamma_{\sigma \rightarrow aa} > \Gamma_{\sigma \rightarrow gg}$ when $\xi \gtrsim 0.05$.

X or \bar{X} may directly couple to the MSSM Higgses if they have non-vanishing PQ charges, which gives the decay mode of saxion into the Higgs boson pair. For example, we may introduce the following term into the superpotential:^{#4}

$$W_{\bar{X}^2 H_u H_d} = \frac{\lambda'}{M_{\text{Pl}}} \bar{X}^2 \hat{H}_u \hat{H}_d, \quad (3.9)$$

where \hat{H}_u and \hat{H}_d are up- and down-type Higgses, respectively. Using the fact that the so-called μ -parameter is generated by the VEV of \bar{X} , we obtain the decay rate as

$$\Gamma_{\sigma \rightarrow hh} = \frac{1}{2\pi} \frac{m_\sigma^3}{F_a^2} \left(\frac{\mu^2}{m_\sigma^2} \right)^4 \left(1 - \frac{m_h^2}{m_\sigma^2} \right)^{1/2}. \quad (3.10)$$

$Br(\sigma \rightarrow hh)$ depends on various parameters. If $\mu \sim O(m_\sigma)$, $Br(\sigma \rightarrow hh)$ and $Br(\sigma \rightarrow aa)$ are of the same order. In models with smaller m_σ (like the gauge mediation), the decay process of $\sigma \rightarrow hh$ is kinematically blocked.

Since we are interested in the case of $F_a \sim 10^{9-12}$ GeV, the lifetime of the saxion becomes very long. If there exists relic saxion in the early universe, the decay of saxion produces axion which survives until today and behaves as an extra radiation component (so-called “dark radiation”) [23]. We will study implications of such a relativistic axion in the present model.

^{#4}If X , instead of \bar{X} , couples to the Higgses, the potential at $X \gg f$ may be modified due to extra thermal effect, which may change the following discussion. We do not consider such a case.

3.2 Thermal Potential

Now, we are at the position to discuss the potential of the saxion. We first comment on the so-called Hubble-induced mass. In the inflaton dominated era before reheating, Planck suppressed interactions of $\hat{\bar{X}}$ and \hat{X} may induce their effective masses of the order of the expansion rate of the universe. Even in radiation dominated universe, this may be the case [26]. (We call such masses as Hubble-induced masses.) The sizes of the Hubble-induced masses crucially depend on the model, and we simply parametrize the potential as^{#5}

$$V_T = \tilde{m}_1^2 |X|^2 + \frac{\tilde{m}_2^2 f^4}{|X|^2} + 2N_C V_T^{(B)}(y f^2/|X|) + 2N_C V_T^{(F)}(y f^2/|X|) - a_L \alpha_3^2(T) T^4 \ln |X|^2 \\ + 2V_T^{(B)}(\lambda \sqrt{|X|^2 + f^4/|X|^2}) + 2V_T^{(F)}(\lambda \sqrt{|X|^2 + f^4/|X|^2}). \quad (3.11)$$

Here, \tilde{m}_i are effective masses which may include the effect of Hubble induced masses; $\tilde{m}_i \simeq m_i$ when $H \ll m_{3/2}$, while $\tilde{m}_i \sim H$ if the Hubble induced masses exist and dominate. In Eq. (3.11), $N_c = 3$ is the color factor. In addition, in our numerical study, we take $a_L = 1$.

Once the amplitudes of \bar{X} and X become non-vanishing, \hat{Q}_0 and \hat{Q}_+ become massive with the mass of $y|\bar{X}|$. In addition, \hat{S} and $\hat{\mathcal{X}}$ acquire the mass of $\lambda\sqrt{|X|^2 + |\bar{X}|^2}$. After the PQ symmetry breaking, the product $\bar{X}X$ is fixed to be $f^2 + O(yT\tilde{m}_1)$. Thus in the situation of our interest, we can eliminate \bar{X} (or X) from the potential. We choose X as the independent variable because \hat{X} plays the role of the axion multiplet when $X \gg \bar{X}$, which is the case in the following discussion.

Although \hat{S} and $\hat{\mathcal{X}}$ also affect the thermal potential, it is instructive to consider the case that the thermal effects are dominated by those of \hat{Q}_0 and \hat{Q}_+ . In such a case, the potential is approximated as

$$V_T \sim \tilde{m}_1^2 |X|^2 + \frac{\tilde{m}_2^2 f^4}{|X|^2} + a_T y^2 T^2 \frac{f^4}{|X|^2} \theta(T - y f^2/|X|) - a_L \alpha_3^2 T^4 \ln |X|^2, \quad (3.12)$$

where a_T is a constant of $O(1)$. When the temperature is high enough, the potential have a minimum at $X \sim x_T$, where

$$x_T \equiv \sqrt{\frac{yT}{\tilde{m}_1}} f. \quad (3.13)$$

(Hereafter, we call the minimum at $X \sim x_T$ as “trapping minimum.”) The trapping minimum disappears when T becomes smaller than the effective mass of the PQ fermions at $X \sim x_T$. This happens when the cosmic temperature is $\sim T_c$, where T_c is the solution of the following equation:

$$T_c = \frac{y f^2}{x_T(T_c)}. \quad (3.14)$$

^{#5}If \bar{X} or X couples to extra fields, contributions of those extra fields should be taken into account. This is the case if, for example, \hat{Q}_0 and \hat{Q}_+ are embedded into complete multiplets of the gauge group of grand unification.

(Notice that, when the Hubble-induced mass is sizable, \tilde{m}_1 depends on background temperature.) On the contrary, at low temperature, the T^2 term in the potential does not exist at the region with $X \ll yf^2/T$. If so, there may exist a minimum at $X \sim x_L$, where x_L is given by

$$x_L^2 \equiv \frac{a_L \alpha_3^2 T^4 + \sqrt{(a_L \alpha_3^2 T^4)^2 + 4\tilde{m}_1^2 \tilde{m}_2^2 f^4}}{2\tilde{m}_1^2}. \quad (3.15)$$

When $a_L \alpha_3^2 T^4 \gtrsim \tilde{m}_1 \tilde{m}_2 f^2$, $x_L \sim a_L^{1/2} \alpha_3 T^2 / \tilde{m}_1$, which can be significantly larger than f . Behavior of the potential at $X \sim x_L$ depends on the effective mass of the PQ fermion at such a field value. If the PQ fermion mass is smaller than T , \hat{Q}_0 and \hat{Q}_+ (and their scalar partners) are thermalized as relativistic particles at $X \sim x_L$, and hence the T^2 term in the potential exists. In such a case, $X \sim x_L$ is not the minimum of V_T , but there exists only one minimum at $X \sim x_T$. This is the case when $T \gtrsim T_*$, where T_* is obtained by solving the following equation:

$$T_* = \frac{yf^2}{x_L(T_*)}. \quad (3.16)$$

(In defining T_* , effects of \hat{S} and $\hat{\mathcal{X}}$ are not included; their effects will be separately considered later). If the Hubble-induced mass is negligible, $T_* = \alpha_3^{-1/3} y^{1/3} m_1^{1/3} f^{2/3}$ (where we have assumed $\alpha_3 y^2 (f/m_{3/2})^5 \gtrsim 1$, which is the case in the situation of our interest.) On the contrary, for $T \lesssim T_*$, the PQ fermions are heavier than T at $X \sim x_L$, and we expect that the potential has a minimum at $X \sim x_L$.

In summary, the thermal potential behaves as follows:

- $T \lesssim T_c$: There is only one minimum at $X \sim x_L$.
- $T_c \lesssim T \lesssim T_*$: There are two minima at $X \sim x_L$ and $X \sim x_T$.
- $T \gtrsim T_*$: There is only one minimum at $X \sim x_T$.

We should also comment here that, if $x_L(T_c) \sim O(f)$, two minima become indistinguishable at $T \sim T_c$. In such a case, trapping minimum, which exists at high temperature, smoothly merges to the zero temperature minimum. Our numerical calculation indicates that this happens when $y \lesssim 7\sqrt{m_2/f}$. In this case, the coherent oscillation of the saxion is not induced.

In Fig. 1, we plot V_T given in Eq. (3.11) for the temperature around $T \sim T_c$, neglecting the effects of \hat{S} and $\hat{\mathcal{X}}$. (Here, we take $\tilde{m}_i = m_i$ and the effects of Hubble-induced masses are omitted.) As one can see, when the temperature is $O(T_c)$, the trapping minimum disappears. In addition, two distinct minima exist just before the disappearance of the trapping minimum. Behaviors shown in the figure are consistent with the previous discussion.

So far, we have considered the case that the thermal effects from \hat{S} and $\hat{\mathcal{X}}$ are negligible. However, if $\lambda f \lesssim T_c$, the potential around $X \sim f$ is deformed. In particular, if λ is sizable,

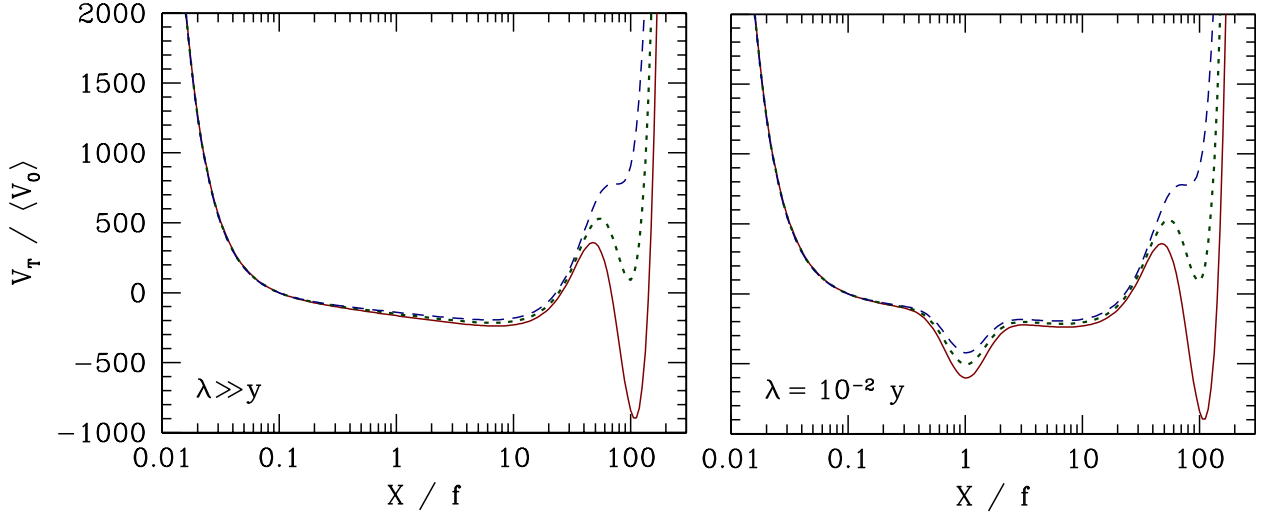


Figure 1: Thermal potential as a function of X/f . We take $f = 10^{10}$ GeV, $m_1 = m_2 = 1$ TeV, and $y = 1$. The solid (dotted, dashed) line is for $T = 0.69T_c$, ($T = 0.67T_c$, $T = 0.65T_c$). We added a constant to the potential. In addition, the vertical axis is normalized by the zero-temperature expectation value of the potential $\langle V_0 \rangle \equiv m_1^2 \langle X \rangle^2 + m_2^2 \langle \bar{X} \rangle^2$. The left figure is for the case that the contributions of \hat{S} and $\hat{\mathcal{X}}$ are negligible, while the right figure is for the case with $\lambda = 0.01$.

there exists a minimum at $X \sim f$ at $T \sim T_*$. In Fig. 1, we also show one of the examples of such a case, taking $\lambda = 0.01y$. Such an extra minimum may affect the evolution of the saxion, in particular the trapping process. (See the following discussion.)

The thermal trapping of the saxion may be cosmologically important, although its implication strongly depends on the details of the model. In the following, we discuss possible effects of the thermal trapping in the present model. We pay particular attention to the energy density of axion produced from the saxion oscillation. If the saxion field is once trapped in the trapping minimum, it starts to oscillate with the amplitude of $\sim x_T$ at $T \sim T_c$. Then, such a saxion oscillation decays with long lifetime. If σ dominantly decays into axion pair, the produced axion survives until today and behaves as dark radiation whose abundance is constrained from the cosmic microwave background (CMB) anisotropy and big-bang nucleosynthesis (BBN).

3.3 Case with $y < \lambda$

Thermal history depends on the mass spectrum of particles in the PQ sector. We first consider the case with $y < \lambda$; in such a case, \hat{S} and $\hat{\mathcal{X}}$ are always heavier than \hat{Q}_0 and \hat{Q}_+ . Then, at $T \lesssim \lambda f$, we neglect the thermal effects from \hat{S} and $\hat{\mathcal{X}}$. The cosmological implication of the saxion depends also on its interactions. Thus, for simplicity, we mostly concentrate on the case that the interaction of the saxion with the Higgses given in Eq. (3.9) is negligible.

We start our discussion with studying the behavior of T_* , which depends on the size of the Hubble-induced mass. In the following study, we consider the case that the Hubble-induced

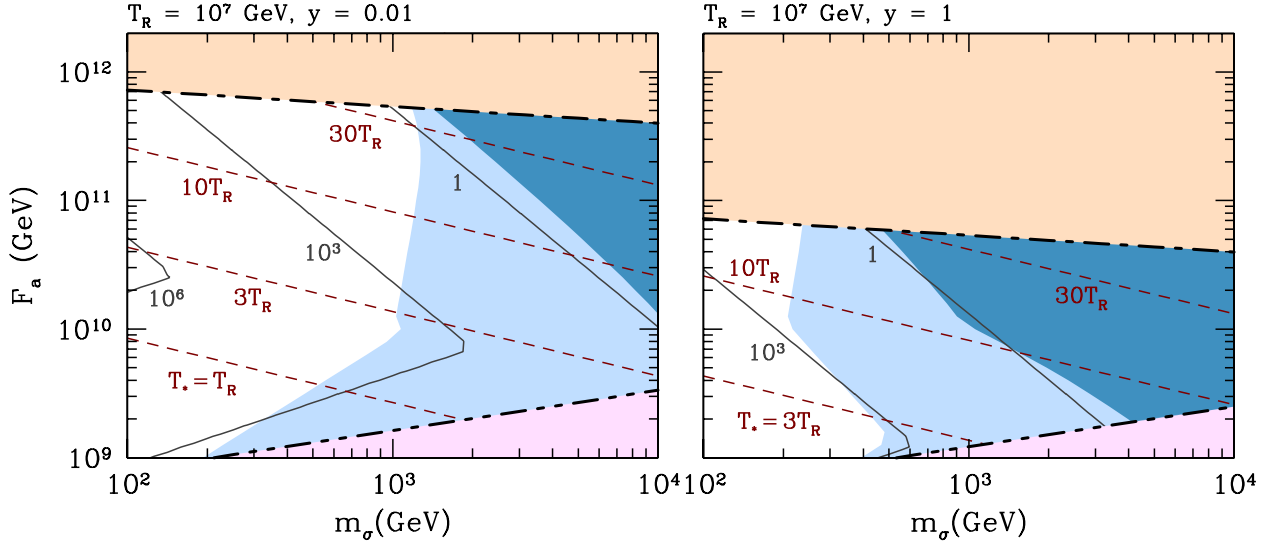


Figure 2: The dotted lines are contours of constant T_* . The solid lines are contours of constant $R_{\sigma/r}$ (see Eq. (3.21)) for $\xi \sim 1$. (Numbers in the figures are the values of $R_{\sigma/r}$.) In the darkly-shaded (dark blue) region, ΔN_ν becomes smaller than 1 if $\Gamma_{\tilde{a}}/\Gamma_\sigma = 10^{-6}$. In the lightly-shaded (light blue) region, ΔN_ν becomes smaller than 1 if the axino decays just before the BBN. Above the dot-dashed line, Eq. (3.16) does not have solution. Below the dot-dot-dashed line, $\Gamma_{\text{diss}} > H$ is realized at a certain epoch after $T = T_c$. Here, $T_R = 10^7$ GeV, and the Yukawa coupling constant is taken to be $y = 0.01$ (left) and 1 (right).

mass is sizable and approximate

$$\tilde{m}_i = \max(m_i, H). \quad (3.17)$$

The Hubble-induced mass relaxes the constraint compared to the case with $\tilde{m}_i = m_i$.

In Figs. 2 and 3, we plot the contours of constant T_* on m_σ vs. F_a plane for fixed values of y and T_R . (Here and hereafter, we approximate $f \sim F_a$ and $m_1 \sim m_2 \sim m_\sigma$.) The behavior of T_* can be understood as follows. For small enough F_a , effect of the Hubble-induced mass is irrelevant in solving Eq. (3.16), and T_* is independent of T_R . On the contrary, for larger value of F_a , effect of the Hubble-induced mass becomes important. Let us consider how T_* behaves when $\tilde{m}_i \sim H$. In the radiation-dominated universe, H is proportional to T^2 , and hence $x_L \simeq a_L^{1/2} \alpha_3 T^2 / H$ becomes insensitive to T . In such a case, T_* , which is the solution of Eq. (3.16), becomes independent of m_1 . On the contrary, in the inflaton dominated universe, $H \propto a^{-3/2}$ while $T \propto a^{-3/8}$, and hence

$$H(T > T_R) \simeq \left(\frac{T}{T_R} \right)^4 H(T_R), \quad (3.18)$$

where T is the temperature of dilute plasma produced by the decay of inflaton. Then, x_L^{-1} is proportional to T^2 as the temperature of dilute plasma increases, and Eq. (3.16) does

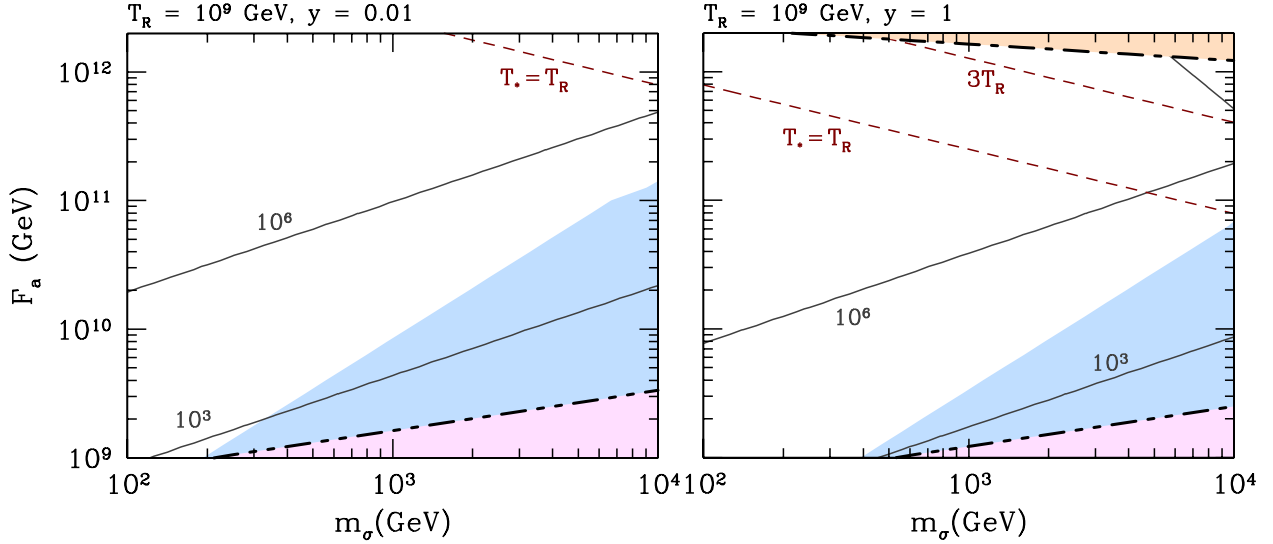


Figure 3: Same as Fig. 2, except for $T_R = 10^9$ GeV. (No dark shaded region on this parameter region.)

not have a solution if F_a is too large. Consequently, the thermal trapping of the saxion is not guaranteed if $F_a \gtrsim g_*^{-3/16} y^{-1/2} \alpha_3^{1/2} m_\sigma^{1/8} T_R^{3/4} M_{\text{Pl}}^{3/8}$. We conservatively assume that no constraint is obtained in such a case.

If the saxion is once trapped in the trapping minimum, the saxion starts to oscillate with the amplitude of $O(x_T)$ when the trapping minimum disappears. Even if X and \bar{X} are of $O(f)$ just after the PQ phase transition, we expect that the thermal trapping occurs if T_* is lower than the maximum temperature of the universe $T_{\text{max}} \sim H_{\text{inf}}^{1/4} M_{\text{Pl}}^{1/4} T_R^{1/2}$, with H_{inf} being the expansion rate during inflation [27].

In Figs. 2 and 3, we also show the region in which the dissipation rate becomes larger than H at a certain cosmic temperature below T_c , where the dissipation rate is evaluated with Eq. (2.10) by replacing F_a with the amplitude of X . In such a region, even if the trapping happens, the relic saxion is dominated by thermally produced one. Notice that the region with $y \lesssim 7\sqrt{m_2/f}$, in which two minima merge smoothly, is mostly covered by the region with $\Gamma_{\text{diss}} > H$. One may think that the scattering processes among the particles in the PQ sector also contribute to the dissipation process. At $T < T_c$, only the particles in the axion multiplet $\hat{\mathcal{A}}$ (i.e., axion, axino, and saxion) can contribute to the dissipation process because the masses of other particles in the PQ sector are larger than T_c . The axion multiplet particles are thermalized only when Γ_{diss} becomes comparable to H . In addition, the interaction of the axion multiplet is suppressed by inverse powers of $F_a^{(\text{eff})}$ because the axion is a Nambu-Goldstone boson. Dissipation rate due to the scattering processes among the PQ sector particles is estimated to be $\sim \frac{1}{4\pi} \frac{T^5}{F_a^{(\text{eff})4}}$ and is smaller than Γ_{diss} given in Eq. (2.10) (with $F_a \rightarrow F_a^{(\text{eff})}$) in the parameter region of our interest.

If the dissipation of the saxion oscillation is negligible, the saxion oscillation with large initial amplitude may result in the overproduction of the axion dark radiation. To see when there may exist such a problem, we calculate the density of the oscillating saxion. The yield variable of the saxion is estimated as

$$Y_{\sigma}^{(\text{osc})} \simeq \frac{45}{\pi^2} \frac{\tilde{m}_{\sigma} \sigma_{\text{R}}^2}{g_*(T_{\text{R}}) T_{\text{R}}^3}, \quad (3.19)$$

where σ_{R} is the saxion amplitude at $T = T_{\text{R}}$, and \tilde{m}_{σ} is the effective mass of saxion which may include the effect of the Hubble-induced mass. In our numerical analysis, we use $g_*(T_{\text{R}}) = 228.75$. (If the saxion starts to oscillate after the reheating, T_{R} should be replaced by the temperature at which the oscillation starts, and σ_{R} should be identified as the initial amplitude of the saxion oscillation.) If the oscillation of the saxion starts before the reheating, $n_{\sigma}(T_{\text{R}})$ is related to the initial number density using the fact that $a \propto T^{-3/8}$ during the inflaton-dominated era. Denoting the temperature at which the saxion starts to oscillate as T_i (with $T_i > T_{\text{R}}$),

$$n_{\sigma}(T_{\text{R}}) \simeq \left(\frac{T_{\text{R}}}{T_i} \right)^8 n_{\sigma}(T_i). \quad (3.20)$$

To see how large the saxion abundance is, we calculate the following ratio

$$R_{\sigma/r} \equiv \frac{\rho_{\sigma}(t_{\text{dec}})}{\rho_r(t_{\text{dec}})}, \quad (3.21)$$

where $\rho_{\sigma}(t_{\text{dec}})$ and $\rho_r(t_{\text{dec}})$ are energy densities of the saxion and the radiation from the inflaton decay at the time just before the saxion decay, respectively. If $R_{\sigma/r} \gtrsim 1$, the saxion dominates the universe. In Figs. 2 and 3, we show contours of constant $R_{\sigma/r}$. We can see that the saxion dominance occurs in large fraction of the parameter space. Then, when it decays, some amount of energetic axion is produced.

The present density of the axion from the saxion decay is parametrized by using the effective number of extra neutrinos as

$$\Delta N_{\nu} \equiv N_{\nu}^{(\text{eff})} - 3 \equiv \frac{3\rho_a(t_{\text{now}})}{\rho_{\nu}(t_{\text{now}})}. \quad (3.22)$$

Here, $\rho_a(t_{\text{now}})$ and $\rho_{\nu}(t_{\text{now}})$ are energy densities of the axion and neutrinos in the present universe, respectively, and the factor of 3 in the numerator is the number of generations of neutrinos. From the latest WMAP observation [28], the best-fit value of the effective number of neutrinos (which includes the standard-model contribution) is obtained as $N_{\nu}^{(\text{eff})} = 4.34^{+0.86}_{-0.88}$ (68% C.L.). In addition, the analysis of ${}^4\text{He}$ mass fraction generated by the BBN reactions indicates $N_{\nu}^{(\text{eff})} = 3.68^{+0.80}_{-0.70}$ (2σ) [29].

If the saxion abundance is too large, ΔN_{ν} exceeds ~ 1 , which conflicts with observations. In estimating ΔN_{ν} , we should notice that the entropy production due to the axino decay

may be sizable in the present case. ΔN_ν becomes smaller as the decay rate of axino $\Gamma_{\tilde{a}}$ gets smaller. The axino decays into gaugino and gauge boson pair and the total decay rate of \tilde{a} is smaller than that of σ if ξ is close to 1. Typically, $\Gamma_{\tilde{a}}/\Gamma_\sigma \sim O(N_i \alpha_i^2 / 2\pi^2 \xi^2)$, where N_i is the number of final states and i depends on the dominant decay mode; if the axino mass is well above the gluino mass, $N_i = 8$ and $\alpha_i = \alpha_3$. However, if the masses of the axino and the gauginos are degenerate, $\Gamma_{\tilde{a}}$ becomes suppressed. So the ratio $\Gamma_{\tilde{a}}/\Gamma_\sigma$ is model dependent. Notice that, if $\Gamma_{\tilde{a}}$ is too small, the axino decays after the BBN, which is likely to spoil the success of the standard BBN scenario. We vary $\Gamma_{\tilde{a}}$ and estimate ΔN_ν . (We use the instantaneous decay approximation.) Here, the axino abundance is evaluated using Eq. (2.9), and the axino mass is taken to be $\frac{1}{2}m_\sigma$. In Figs. 2 and 3, we show the region in which ΔN_ν becomes smaller than 1 if $\Gamma_{\tilde{a}}/\Gamma_\sigma = 10^{-6}$, and also the region in which ΔN_ν becomes smaller than 1 if the axino decays just before the BBN. If $\Gamma_{\tilde{a}}/\Gamma_\sigma = 10^{-6}$, $\Delta N_\nu \gtrsim 1$ for $R_{\sigma/r} \gtrsim 1$ even with the entropy production due to the axino decay. If the axino decays just before the BBN, the effect of the entropy production is more significant. Here, the energy density stored in the axino sector is assumed to be fully converted to that of radiation after the axino decay. This may be due to the pair annihilation of the LSP or due to the decay of the LSP via R -parity violation. Even with such an extreme assumption, ΔN_ν may be larger than ~ 1 if $Br(\sigma \rightarrow aa) \simeq 1$ in significant fraction of the parameter space.

Such a problem may be avoided if the SUSY breaking parameters are tuned so that the ξ parameter becomes relatively small. Indeed, if $\xi \lesssim 0.05$, $Br(\sigma \rightarrow gg)$ becomes comparable to or larger than $Br(\sigma \rightarrow aa)$, and the production of the axion is suppressed. In the region with $R_{\sigma/r} \gtrsim 1$ of the figure, saxion once dominates the universe. Even so, $\Delta N_\nu \lesssim 1$ is possible if $Br(\sigma \rightarrow aa) \sim O(0.1)$. In fact, as one can see, the constraints on $N_\nu^{(\text{eff})}$ from the CMB anisotropy and BBN indicate non-vanishing value of ΔN_ν ; $\Delta N_\nu \sim 1$ is preferred. If $Br(\sigma \rightarrow aa) \simeq 0.25$, $\Delta N_\nu = 1$ is realized (with $g_* = 100$ at the time of saxion decay). Another solution is to introduce the superpotential interaction given in Eq. (3.9). With such an interaction, σ may dominantly decay into Higgs boson pair, and $Br(\sigma \rightarrow aa)$ can be $O(0.1)$.^{#6} In such a case, the saxion may also decay into the Higgsino pair, which may result in the overproduction of relic LSP. However, such a difficulty can be avoided with large enough pair annihilation cross section of the LSP. This is the case, for example, if the LSP is the neutral Wino [18]. If the saxion dominantly decays into MSSM particles, one should take account of the entropy production due to the saxion decay. The dilution factor is $\sim R_{\sigma/r}^{3/4}$.

3.4 Case with $y > \lambda$

Next, we consider the case with $y > \lambda$. In such a case, if $X \sim \bar{X} \sim f$, \hat{Q}_0 and \hat{Q}_+ become heavier than \hat{S} and $\hat{\mathcal{X}}$. Even so, the trapping may happen if $T_* \lesssim \lambda f$; in such a case, discussion in the previous section holds.

^{#6}The interaction with the Higgses may enhance the dissipation rate, which may also help to reduce the density of the axion. The study of the dissipation rate in such a case will be given elsewhere [30].

If $T_* \gtrsim \lambda f$, the situation is rather complicated. This is because \hat{S} and $\hat{\mathcal{X}}$ may be fully thermalized as relativistic particles at $T \lesssim T_c$. Then, the scattering processes with \hat{S} and $\hat{\mathcal{X}}$ significantly contributes to the dissipation of the oscillation of the saxion. In addition, if $T_* \gtrsim \lambda f$, there may exist a minimum at $X \sim f$ caused by the thermal effects of \hat{S} and $\hat{\mathcal{X}}$, with which the trapping process to the trapping minimum may not happen. (See Fig. 1.) The \hat{S} and $\hat{\mathcal{X}}$ can be produced from the scattering of particles in thermal bath (like gluon). Even though the interaction between the MSSM sector and the PQ sector (which consists of \hat{X} , $\hat{\bar{X}}$ and \hat{S}) may be too weak to equate the temperatures of two sectors, we expect that the interactions among \hat{X} , $\hat{\bar{X}}$ and \hat{S} are strong enough to thermalize the PQ sector; such a thermalization occurs through the superpotential interaction of $\lambda \hat{S} \hat{\bar{X}} \hat{X}$. Thus, if the masses of \hat{S} and $\hat{\mathcal{X}}$ are smaller than T_* (at $X \sim f$), constraint obtained in the previous section may not be applicable. This is the case when $\lambda \lesssim y^{1/3}(m_\sigma/F_a)^{1/3}$.

3.5 Saxion from Scattering of Thermal Particles

Finally, we comment on the effects of thermally produced saxion. As we have mentioned, scattering processes of thermal particles produce saxion. In particular, if $\Gamma_{\text{diss}} \sim H$ is realized at some epoch, the saxion are thermalized. In such a case, the overproduction of the axion may occur if the saxion dominantly decays into axion pair. However, the abundance of thermally produced axino is comparable to that of saxion. Then, entropy production due to the decay of axino (as well as due to the subsequent pair annihilation of the LSP) dilutes the axion abundance. If the decay rate of the axino is smaller than that of saxion, the effect of the dilution is large enough to make ΔN_ν smaller than ~ 1 .

4 Discussion

In this paper, we have studied effects of saxion on the evolution of the universe paying particular attention to thermal effects on the saxion. Because the axion multiplet necessarily couples to colored particles (i.e., PQ fermions), saxion potential is inevitably deformed at high temperature. In addition, the coherent oscillation of the saxion may dissipate via the interaction with hot plasma.

Taking account of these effects, we have studied the constraint on the supersymmetric PQ model with the superpotential given in Eq. (3.1). Because of the thermal effect on the potential, the saxion field may be trapped in a false minimum before the start of the oscillation, which may result in an overproduction of relativistic axion. The constraint strongly depends on the model parameters. We have seen that, if no PQ sector particle remains in thermal bath after the onset of the oscillation, $\Delta N_\nu \gg 1$ may happen if $Br(\sigma \rightarrow aa) \simeq 1$. In addition, if the dissipation rate is large enough, the saxion oscillation dissipates soon after the start of the oscillation.

In this study, we have concentrated on the model with the superpotential given in Eq. (3.1). However, the thermal effects may have significant effects in other classes of models.

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